# Physically plausible K-space trajectories for Compressed Sensing in MRI: From simulations to real acquisitions

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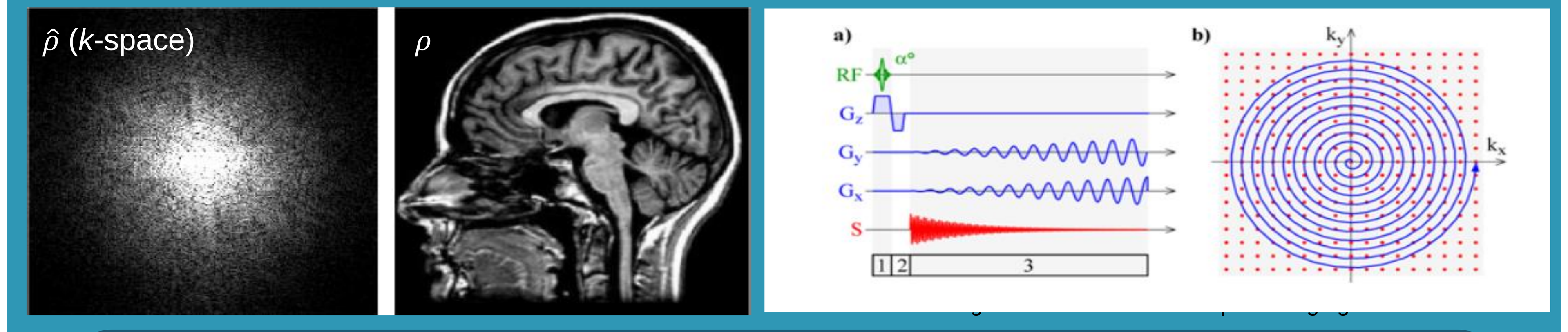
## Abstract

Magnetic resonance imaging (MRI) is a medical imaging technique used in radiology to image the anatomy and function of the body in both health and disease. MRI is probably one of the most successful application fields of compressed sensing (CS). Despite recent advances, there is still a large discrepancy between theories and actual applications. Overall, many important questions related to sampling theory remain open. In this work, we address one of them: given a set of hardware constraints (e.g. sampling Fourier coefficients along smooth curves), how to optimally design a sampling pattern? We first derive three key aspects that should be carefully designed by inspecting the literature, namely *admissibility*, *limit of the empirical measure* and *coverage speed*. To fulfill them jointly, we then propose an original approach which consists of projecting a probability distribution onto a set of admissible measures. The proposed algorithm allows to handle arbitrary hardware constraints (gradient magnitude, slew rate) and then automatically generates efficient sampling patterns. The MR images reconstructed using the proposed approach have a significantly higher SNR (2-3 dB) than those reconstructed using more standard sampling patterns (e.g. radial, spiral), both for medium and very high resolution imaging. Likewise, reconstructions from highly undersampled data acquired in experiments performed on a 7T SIEMENS MR scanner show the superiority of our sampling schemes over traditional MR samplings and proved that very large acceleration factor (up to 40-fold) are practically achievable with CS-MRI.

## Sampling in Magnetic Resonance Imaging

Displacement in k-space is performed via magnetic field gradients.
At time t, the k-space position $s(t)$ and gradient waveform g(t) are related by:
where $\gamma$ denotes the gyromagnetic ratio.

$$s(t) = s(0) + \gamma \int_0^t g(\tau)d\tau$$




### Sampling/Hardware constraints in MRI

The constraints on $g$ read: $\|g(t)\| \leq G_{\max}$, $\|\dot{g}(t)\| \leq S_{\max}$
The values of the bounds $G_{\max}$ and $S_{\max}$ are specified by the gradient system manufacturer. Then, the set of admissible sampling curves is defined by:

$$\mathcal{S}_{MRI} = \{s \in (\mathcal{C}^2([0,T]))^d, \|\dot{s}(t)\| \leq \gamma G_{\max}, \|\ddot{s}(t)\| \leq \gamma S_{\max}, \forall t \in [0,T]\}$$

**Objective**: Minimize the acquisition time $T_\varepsilon$ such that there exists $g: [0, T_\varepsilon] \to \mathbb{R}^d$ satisfying the constraints by collecting the samples $E(s) = \{\hat{\rho}(s(k\Delta t))\}_{k \in \{0,\cdots,T_\varepsilon/\Delta t\}}$ along curve $s$ that allows reconstructing $\tilde{\rho}$ such that $\|\rho - \tilde{\rho}\| \leq \varepsilon$.

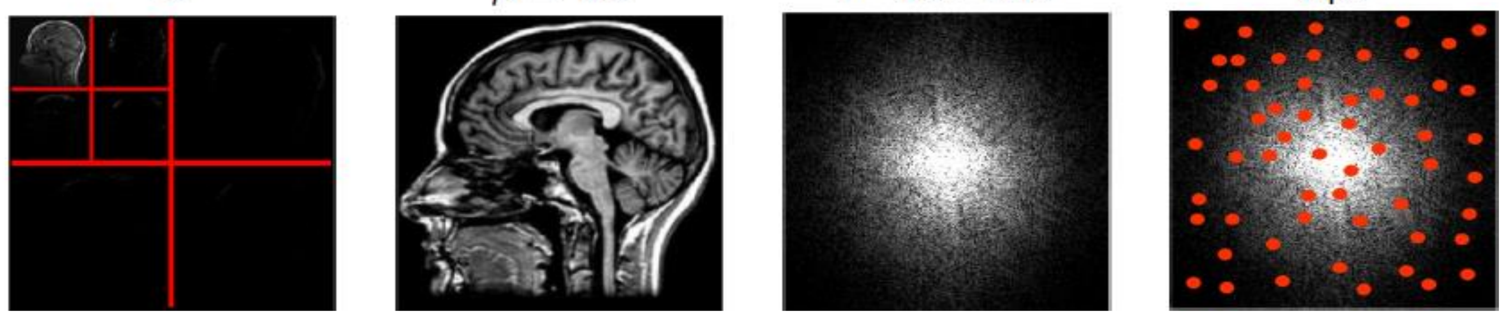
1. *How to choose the measurements $E(s)$?*
2. *How to find optimal $s$?*

## Compressed Sensing in MRI

- $\rho$ is sparse in a given basis (e.g. wavelets), $\rho = \Psi x$, where $x \in \mathbb{C}^n$ is $s$-sparse.
- Acquisition matrix: $A = F^*\Psi$.

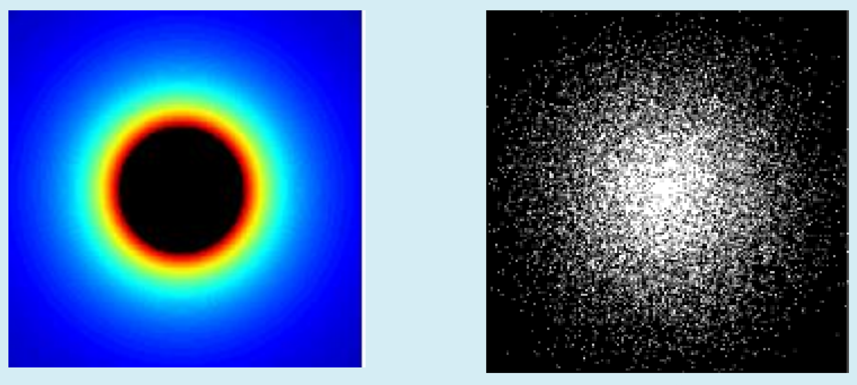
Let $x \in \mathbb{C}^n$ denote an $s$-sparse representation of the image.
Let $\Gamma \subseteq \{1, \cdots, n\}$ and $A_\Gamma = (a_i^*)_{i \in \Gamma}$. We acquire a measurement vector:

$$y = A_\Gamma x.$$




### Variable Density Sampling (VDS)

- Sample more frequently the low frequencies (center of K-space)



Radial VDS

*$s \notin \mathcal{S}_{MRI}$*
*This is not feasible!*

### Nonlinear reconstruction

$$\underset{z}{\text{minimize}} \overbrace{\|Az - y\|_2^2}^{\textit{Data consistency}} + \underbrace{\lambda \|z\|_1}_{\textit{Enforces sparsity}}$$

$F$ : NFFT
$\psi$ : sparsifying transform
$A = F\psi^{-1}$
$y$ : acquired data
$x$ : image
$z = \psi x$ : sparse representation of x
$\lambda$ : regularization parameter

## Design of feasible gradient waveforms: A Projection Problem on Measure set

### The general construction (a discretized problem)

- Approximate $\mathcal{M}(\mathcal{P})$ by a subset $\mathcal{N}_p \subset \Omega^p$ of $n$-point measures:

$$\mathcal{N}_p = \mathcal{M}(\mathcal{Q}_p) = \left\{ \nu = \frac{1}{p}\sum_{i=1}^{p} \delta_{q_i}, \quad \text{for } q = (q_i)_{1 \leq i \leq p} \in \mathcal{Q}_p \right\},$$

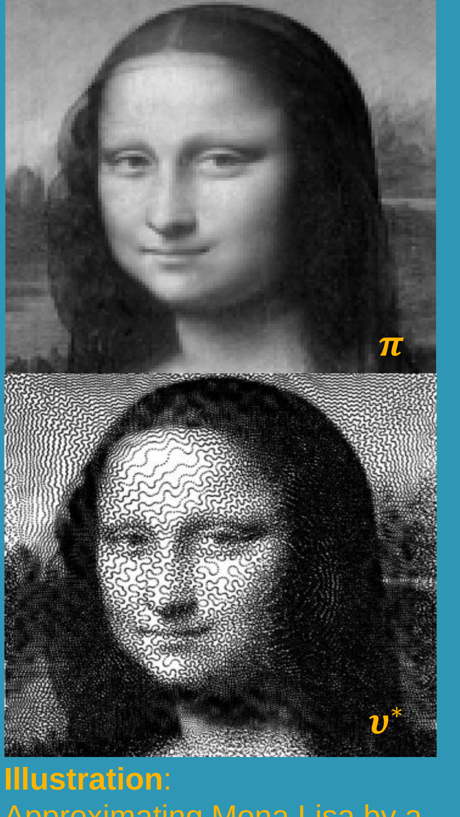
  where $\mathcal{Q}_p$ is the discretized version of $\mathcal{P}$.
- Use a projected gradient descent to obtain an approximate projection $\nu_p^*$ on $\mathcal{N}_p$:

$$\nu_p^* \in \underset{\nu \in \mathcal{N}_p}{\operatorname{Arg\,min}} \frac{1}{2} \|h \star (\nu - \pi)\|_2^2,$$

- Reconstruct an approximation $\nu \in \mathcal{M}(\mathcal{P})$ from $\nu_p^*$.
- $\min_{\nu \in \mathcal{N}_p} \frac{1}{2} \|h \star (\nu - \pi)\|_2^2 =$

$$\min_{q \in \mathcal{Q}_p} J(q) = \underbrace{\frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{p} H(q_i - q_j)}_{\text{Repulsion potential}} - \underbrace{\sum_{i=1}^{p} \int_\Omega H(x - q_i) d\pi(x)}_{\text{Attraction potential}},$$

- *Specific projection algorithm: $P_{\mathcal{Q}_p}$ [Chauffert et al, 2016b]*
- *Gradient computation by fast summation using the NFFT library [Potts & Steidl, 2003]*

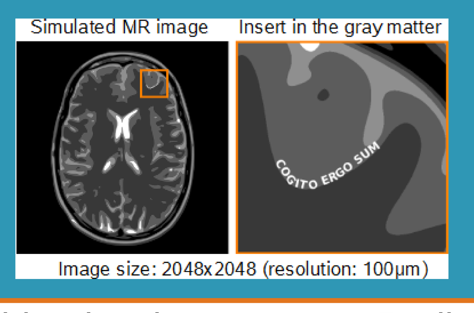



[Chauffert et al, 2016a]

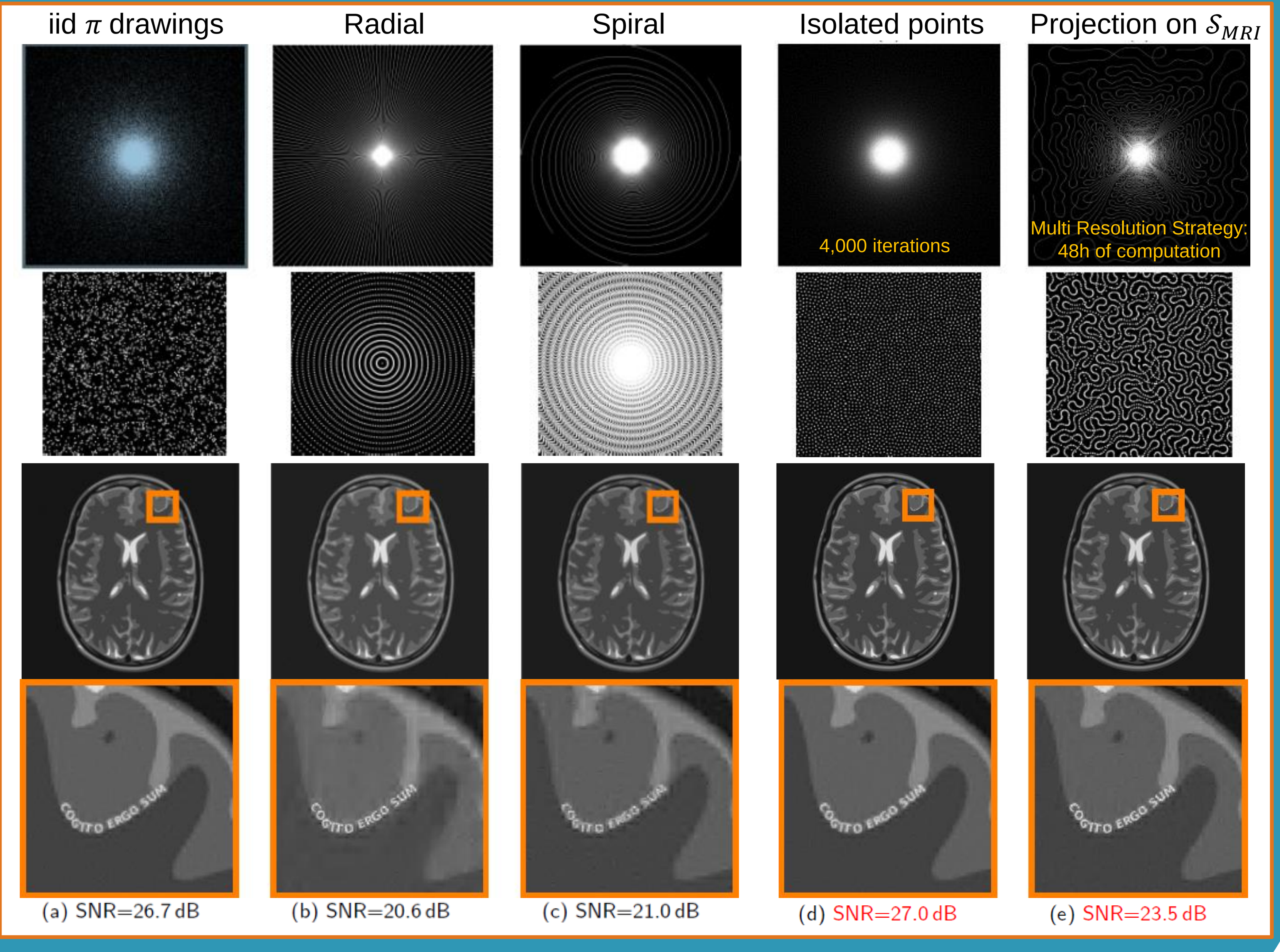
**Illustration:**
Approximating Mona Lisa by a spaghetti i.e. by projecting onto the set $\mathcal{S}_{MRI}$ ($p = 100{,}000$) after 10,000 iterations

## Application to Design of *k*-space Trajectory

**Numerical resolution on 32 core CPU@2.4 GHz (RAM 192 Gb)**




$n = 2048 \times 2048$, $G_{\max}$=40 mT.m[-1], $S_{\max}$ = 150 mT.m[-1].ms[-1]
Only 4.8% of full *k*-space data were sampled ($m = 200{,}000$). For projection, we used $p = 25{,}000$ points/curve ($T = 200$ms) and 8 curves in total:
- **20-fold acceleration compared to whole k-space acquisition**

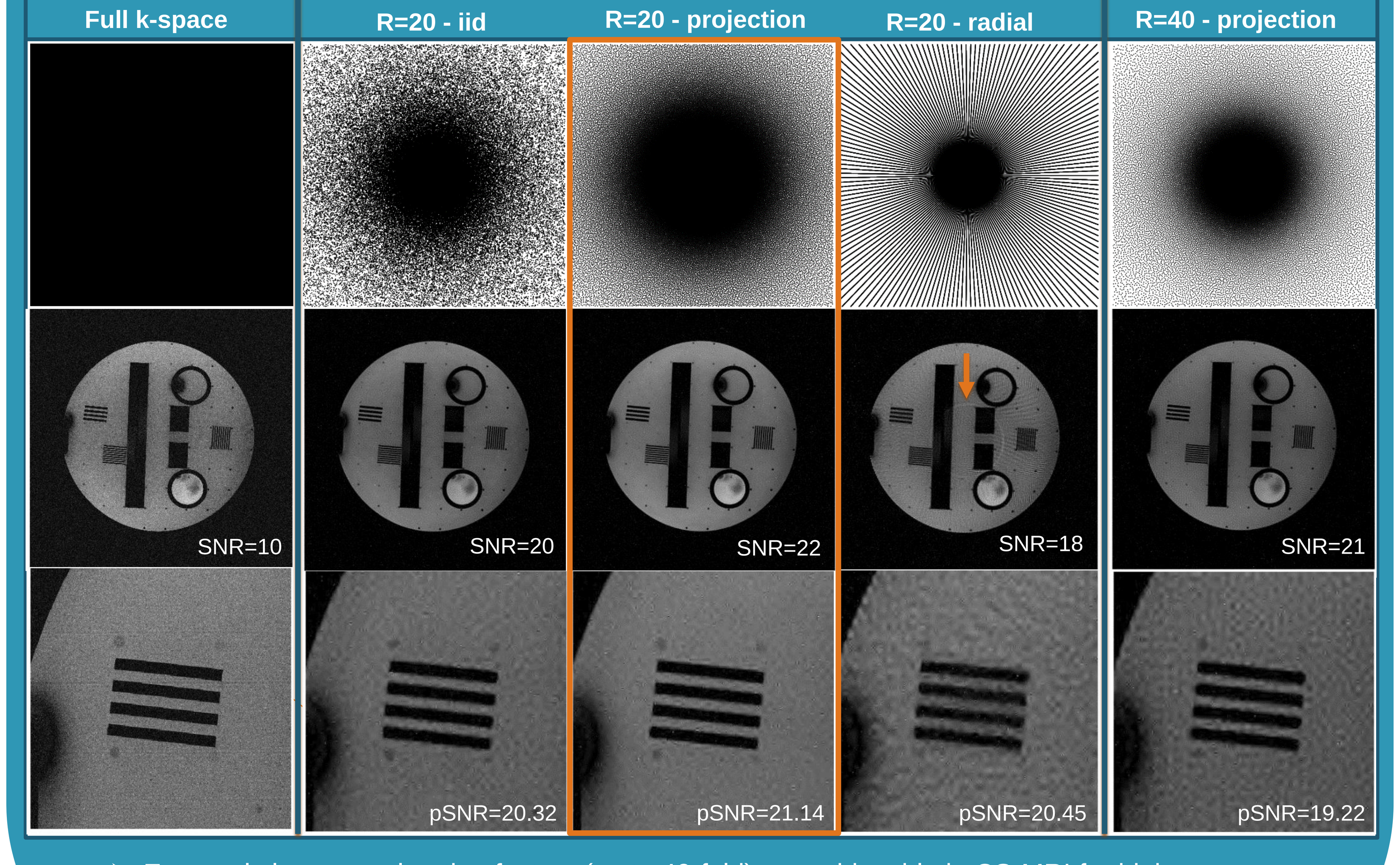



- Projection on measures brought by curves in $\mathcal{S}_{MRI}$ outperforms radial and spiral imaging by 2 to 3 dB

**[Chauffert et al, 2016c]**

## Image reconstructions from undersampled MR data

- Very high resolution CS-MRI images: 0.120 x 0.120 x 5 $mm^3$ – Matrix size: 2048x2048
- Pointwise acquisition inspired by spectroscopic MRI (T1W), perfomed on 7T SIEMENS MR scanner
- Non-linear reconstruction performed with FISTA algorithm ($\lambda = 10^{-4}$)




- Extremely large acceleration factors (up to 40-fold) are achievable in CS-MRI for high resolution imaging
- MR acquisitions show superiority of projection sampling schemes over radial or iid drawings

**(Unpublished work by C. Lazarus, A. Vignaud, P. Ciuciu)**

## Conclusions

In this work, we have proposed an original computer-intensive approach to design efficient sampling schemes complying with the hardware constraints of MRI gradient systems. On the reconstructed images we have shown significant improvements in terms of image quality (pSNR) in very high resolution anatomical imaging, which makes sense for in-vivo exams at ultra-high magnetic field (ISEULT Project@NeuroSpin). MR acquisitions on 7T MR scanner showed superiority of developed sampling schemes and suggest the feasibility of very high acceleration factor at very high resolution in CS-MRI.

## References

[Chauffert et al, 2016a]: **A projection algorithm on measures sets.** *Accepted in Constructive Approximation,* 2016. [Chauffert et al, 2016b]: **A projection algorithm for gradient waveform design in Magnetic Resonance Imaging.** *Accepted in IEEE Transactions on Medical Imaging,* 2016. [Chauffert et al, 2016c]: **On the generation of sampling schemes for Magnetic Resonance Imaging.** Submitted to *Applied and Computational Harmonic Analysis*, 2016. [Lustig et al, 2007]: *Magnetic resonance in medicine* 58.6 (2007): 1182-1195. [Pott & Steidl, 2003]: *SIAM Journal on Scientific Computing*, 24(6): 2013-37, 2003.